DOCUMENT RESUME

ED 348 387 TM 018 787

AUTHOR Kreft, Ita G. G.
TITLE The Analysis of Small Group Data: A Reanalysis of Webb 1982 with a Random Coefficient Model.
PUB DATE Apr 92
NOTE 16p.; Paper presented at the Annual Meeting of the American Educational Research Association (San Francisco, CA, April 20-24, 1992).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Groups; Junior High Schools; *Junior High School Students; *Regression (Statistics); *Reliability; Research Utilization; *Sample Size; Small Group Instruction
IDENTIFIERS *Hierarchical Linear Modeling; Linear Models; *Random Effects; VARCL Computer Program

ABSTRACT

The analysis of small group data with hierarchical linear models is discussed, concentrating on the usefulness and reliability of such analyses using data reported by N. M. Webb (1982). Results of Webb's analyses for 96 junior high school students in small groups are compared with results obtained with random effects linear models for the analysis of hierarchically nested data with the VARCL computer package. It is concluded that the traditional linear model used by Webb produces regression estimates that are very close to the ones produced by random effects models. Based on these results, it is argued that the use of random effects linear models does not always produce the strikingly different results that some applications of these techniques suggest, as shown by S. H. Raudenbush and D. J. Willms (1991). The regression coefficients of fixed effect linear models are fairly robust in data analysis of small groups, even when assumptions of the fixed effects linear model are clearly violated. The difference between fixed and random effects linear models is mainly in the opportunities to test more complicated models with the latter. The difference between fixed versus random models is not in the estimates of the regression coefficients; it is in the conclusions reached. Using the same data set, different conclusions are reached than those drawn by Webb. Choosing between traditional fixed effects linear models and random effects linear models is discussed. Four figures, 3 tables, and a 21-item list of references are included. (Author/SLD)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED348387

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
☒ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

THE ANALYSIS OF SMALL GROUP DATA: A REANALYSIS OF WEBB 1982 WITH A RANDOM COEFFICIENT MODEL.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
ITA G.G. KREFT
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

Ita G.G. Kreft.
California Sate University, Los Angeles
School of Education
Division of Educational Foundations and Interdivisional Studies.

**Paper to be presented at the Annual meeting of the AERA,
San Francisco, April 20-24, 1992
Session title: "Application of Statistical Modeling to Educational Problems.**

With special thanks to Dr. Noreen M. Webb, for allowing me to access her data.

Abstract.
The analysis of small group data with hierarchical linear models is discussed. The discussion concentrates on the usefulness and reliability of such analyses using data reported in Webb (1982). The results of Webb's analysis are compared with the results obtained with random effects linear models for the analysis of hierarchically nested data with the package VARCL (Longford, 1987). We reach the conclusion that the traditional linear model used by Webb produces regression estimates that are very close to the ones produced by random effects models. Based on these results we argue that the use of random effects linear models does not always produce the strikingly different results as some applications of these techniques suggest (see e.g. Raudenbush and Willms,1991). The regression coefficients of fixed effect linear models show to be fairly robust in data analysis that consist of small groups, even when assumptions of the fixed effects linear model are clearly violated. We show that the difference between fixed and random effects linear models is mainly in the opportunities to test more complicated models with the latter. The difference between fixed versus random models is not in the estimates of the regression coefficients, it is in the conclusions reached. We show that, based on the same data set, we reach different conclusions than Webb (1982) as a direct result of the choice for a random effects linear model. We present arguments why and how a researcher of small group dynamics can make a choice between traditional fixed

TM018787

BEST COPY AVAILABLE

effects linear models and the newly developed random effects linear models.

1. Assessment of the problem.

Results of small group studies show interaction effects of 'attitude by treatment' (Webb 1982). According to Cronbach and Webb (1975) the presence of such interactions makes straightforward generalization of results questionable, when these are based on analyses ignoring such interaction effects. A translation of 'attitude by treatment' effect for our data is assuming that: the negative effect of 'asking a question and receiving no answer' on the achievement of an individual student is stronger if a student works with other students in homogeneous groups than if a student works in a heterogeneous groups. In more general terms: it is assumed that variable relationships exists between the dependent and independent variables across groups. The research problem investigated in this paper follows closely Webb's (1982, p.476) research question, which is to establish the different effects of relationships between students' interaction and their achievement in small mixed-ability versus uniform -ability groups. This can be achieved by fitting a linear regression model, while assuming that regression models are different across small learning groups. For instance, the slope coefficient for type of interaction (X or X's) an individual experiences within his learning group, has an effect on individual student achievement (Y), which varies from group to group. This variation across groups is modeled as a function of the characteristic of the group (either mixed ability or uniform ability), which leads to the notion of multilevel analysis. The *multi*-levels are in our example: the level of the student and the level of the learning group.

From the assumption of heterogeneous relationships between X and Y across groups follows that different regression models need to be fitted for different groups. In data that contains of small groups the fitting of a model in each group separately is close to impossible as a result of the small number of observations. Besides the problem of sample size we may encounter another problem: the variable may have a constant value within the group, which per definition is true when a group contains only a single observation. In our example groups of one observation exist, resulting from missing data.

For all reasons given sofar, the analysis method of separate models for separate groups (see Burstein, Linn and Capell, 1978) is not feasible Since 1984 (e.g. Mason, Wong and Entwissle) analyses methods are developed based on the same principles; separate models for separate groups. However the new models are different in one important feature; the analysis involves the total sample, where groups are analyzed in relation to each other. Groups are not considered as separate and unrelated samples, but as replications of each

other and analyzed together in a single analysis. This model is known under different names, such as 'The Hierarchical Linear Model' (HLM, 1988), 'Variance Component Analysis' (VARCL, 1986) and 'Random Coefficient Model' (DeLeeuw and Kreft, 1986). In this paper we refer to this model as random effects or random coefficient (RC) model. The problems in fixed effects analysis models with small group data, are no longer relevant for RC models. Groups with constant X (or X's), and groups consisting of one single observation, do not prohibit an, nor do we have to exclude these groups. The only condition that has to be fulfilled is, that the data has to contain at least two groups with non constant values for X, since RC models are based on empirical Bayesian methodology which facilitates the borrowing of information across groups. The question of generalizability of results based on RC models is the question of generalization of aptitude/treatment interactions to other group settings.

Small sample properties of estimates based on RC models are explored in other setting than ours. Since small numbers are in general related to unreliable estimates for the coefficients in a linear model, a common opinion is, that RC models need larger groups and large total numbers of observation in order to produce reliable estimates. We base latter conclusion on research with artificial data sets, that investigate small sample properties of the estimates of the hierarchical linear model (Bassiri, 1990), ideal sample sizes for multilevel data analysis (Kim, 1990), and the accuracy of standard errors in relation to sample sizes (Snijders and Bosker, 1992). The conclusion based on this literature is that larger groups produce more reliable estimates, while a trade off exists between the total number of observations and the total number of groups. Non of this literature deals with small groups of two, three or four observations, as we have in our example. General guidelines for optimal numbers of observations are hard to give. It depends on a number of factors and combination of factors. Except the number of observations in total, the number of groups is important as well as the number of variables included in the model, the number of random coefficients, number of interactions, and the value of the intra class correlation. For example, a small intra class correlation, say $r=0.10$ means that you loose approximately 10% of the total number of observations if a simple model is fitted with only a random intercept. As soon as random slopes are part of the model the loss of independent observations becomes more complicated to calculate.

To establish the reliability of the results with such small group data, we compared results of RC models with results of equivalent linear models with fixed effects. If results of our real life example shows that RC models produce reliable estimates, our analysis may have implications for future use of RC models in research with small group data.

The data is collected and the research problem is formulated and investigated by Webb

(Webb, 1982, Webb and Cullian, 1983, and Webb, 1984). These studies investigate interaction and achievement in cooperative small groups in four junior high school mathematics classrooms. Groups consist of one to four students and are characterized as either mixed-ability or uniform-ability learning groups. The way students interact with each other in the learning groups is hypothesized to influence individual student achievement. Webb 's (1982) study explores the function of group interaction as mediating variable between group and individual characteristics and achievement (l.c. p.476).

2. More about multilevel analysis for small group data.

Multilevel software allows the researcher to measure accurately the relationships between observations within and between groups at each level of the hierarchy. A seemingly complex statistical task made much easier by the advent of multilevel techniques and subsequent software (GENMOD, 1988; HLM, 1988; ML3, 1989; and VARCL, 1986), for the analysis of hierarchical nested data with two or more levels of nesting. The model on which this software is based is a linear regression model with random instead of fixed parameters. The coefficients of the micro or first level variables (in our example characteristics of students) are allowed to vary randomly over groups, while macro or second level variables (in our example the characteristics of small cooperative learning groups) are allowed to interact with the coefficients of the first level. Such interactions are called cross-level interactions.

Some of the assumptions on which RC models are based are different from the fixed effects model, some are equal. An assumption in which fixed and random effects models differ is that in the latter individual error terms are assumed to be correlated between observation within the same group or context. The resulting intra class correlation is defined in RC models as the ratio of the between group variation and the total variation. The less different groups are the lower the intra class correlation. If groups differ extensively the intra class correlation is large. Another assumption in which the RC model differs from the traditional linear model is the assumption that groups are a sample of all possible groups, hence random instead of fixed effects. Webb's data are a good example to illustrate the differences between the models. Students are nested within small cooperative learning groups. The groups are formed within existing school classes The formed groups are clearly a sample of all possible groups that can be formed in all possible schools. Results can be generalized to other settings in other schools. For the data analysis we use one of the multilevel techniques available (VARCL), where the unit of analysis is students within small groups.

3. Description of the data

If we delete cases with missing data 96 out of the original 105 students are left. They are divided over 35 groups. These groups are: 23 groups of 3 students (66%), 9 groups of 2 students (26%), 2 groups of 4 students (6%) and 1 group of 1 student. Assignment of students to groups are done based on CTBS scores obtained at the beginning of the school year, where students are defined as high (top 25%), medium (middle 50%) and low (bottom 25%) see Webb and Cullian (1983, p.415). Uniform-ability groups consisted of students from the same stratum; mixed-ability groups consisted of students from more than one ability stratum.

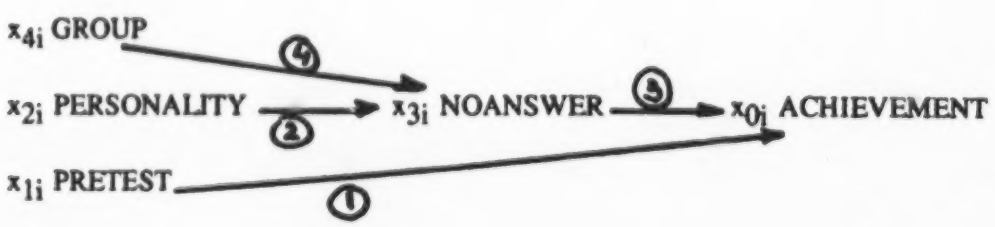
Variables used in this study are four student level variables and one context level variable, all based on Webb (1982). The dependent student level variable is math achievement measured after a period of small group learning. The predictors are: a pretest score (pretest), a personality score, which locates each student on an introversion-extroversion scale (Personality) and the number of times the student asked a question and did not receive an answer (NoAnswer). For more details see Webb (1982, Pp. 476-478). The predictor at the group level is the standard deviation of the pretest scores, calculated for each group separately. To make this last variable comparable to the one used by Webb, homogeneous groups are defined as having a standard deviation on the pre-test equal to, or smaller than 0.75, remaining groups are categorized as mixed ability. We obtained 14 uniform groups (coded as 1) and 21 mixed ability groups (codes as 2). This way we obtained the same numbers of groups as reported in Webb (1982).

4.Webb's analysis results reproduced.

4.1. Multilevel regression with context variable.

Webb's conclusions are mainly based on the student level path analysis model in Figure 1, where each path represents a direct effect of an independent variable on a dependent one.

Figure 1. Webb's path model



The group level variable (GROUP) in this model is dichotomous, and functions as a design variable. The model is reproduced by fitting two separate equations. In the first equation Achievement is predicted by Noanswer and Pretest. In the second equation Noanswer is predicted by Group and Personality (see I and II).

I. $\underline{x}_{0i} = \alpha_0 + \beta_{01}x_{1i} + \beta_{03}\underline{x}_{3i} + \varepsilon_{0i}$
II. $\underline{x}_{3i} = \alpha_3 + \beta_{32}x_{2i} + \beta_{34}x_{4j} + \varepsilon_{3i}$

Variables in above and following equations are defined as: x_{0ij} the dependent variable Achievement, x_{1ij} Pretest , x_{2ij} Personality test, x_{3ij} # questions asked and not answered (NoAnswer), and x_{4j} (Z_j) the group level variable (uniform or mixed ability group). Note that in aggregated form we write Z_j for the group variable x_{4j} in equation II. In the following RC models Z_j (or x_{4j}) is treated differently than in the Webb analysis. Z_j is treated as a group level variable, instead of as a disaggregated student level variable. Note that the notation of the variables in the following equations is different from the notation used in above. For each student variable a subscript j is added to indicate the group membership of the student. Group effects are explicitly introduced in the following models.

We replicate Webb's path analyses in Figure 1 again by fitting the two equations, but now within the multilevel framework of the RC model. The main reason for using a RC model instead of a fixed effect model is to take care of the clustering effect by assuming an intra class correlation and random treatment groups. It also allows us to test for cross level interactions. A possible cross level interaction we will test here is the by Webb suggested varying effects over uniform and mixed ability groups of the relation NoAnswer and student achievement.

The following two equations reproduce the Webb path model in a RC framework. We adopt the convention to underline random coefficients. Equation (1) shows a regression with achievement as the dependent variable and NoAnswer and Pretest as the student level predictors. The second analysis (see equation 2) has NoAnswer as the dependent variable and Personality (introversion versus extroversion) as the student level predictor and Group as the group level predictor. At the macro level the variable for 'Group' is introduced as Z_j.

The micro level equations are in (1a and 2a) and the macro equation in (1b and 2b)

$$\underline{x}_{0ij} = \underline{\alpha}_{0j} + \beta_{01}x_{1ij} + \beta_{03}\underline{x}_{3ij} + \varepsilon_{0ij} \qquad (1a)$$

where

$$\underline{\alpha}_{0j} = \gamma_{00} + \underline{\delta}_{0j} \qquad (1b)$$

Substitution results in (1c)

$$x_{0ij} = \gamma_{00} + \beta_{01}x_{1ij} + \beta_{03}\underline{x}_{3ij} + \delta_{0j} + \varepsilon_{0ij} \quad (1c)$$

and

$$\underline{x}_{3ij} = \alpha_{3j} + \beta_{32}x_{2ij} + \varepsilon_{3ij} \quad (2a)$$

where

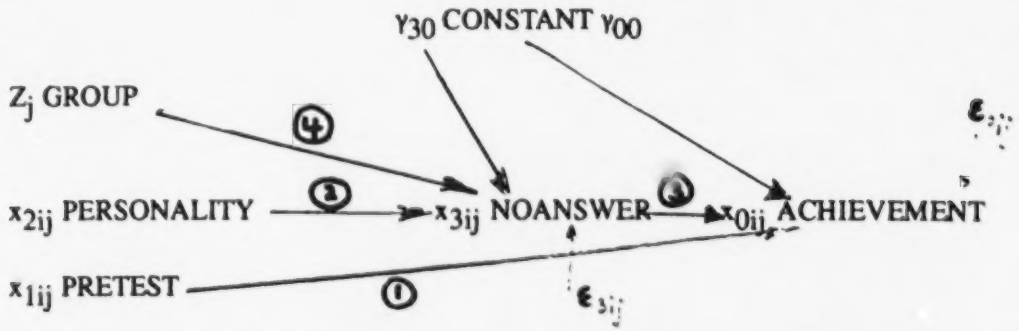
$$\alpha_{3j} = \gamma_{30} + \gamma_{31}Z_j + \delta_{3j} \quad (2b)$$

Substitution results in (2c)

$$\underline{x}_{3ij} = \gamma_{30} + \beta_{32}x_{2ij} + \gamma_{31}Z_j + \delta_{3j} + \varepsilon_{3ij} \quad (2c)$$

The resulting path models is specified in Figure 2

Figure 2



Results of equations (1) and (2), compared with results reported by Webb (1982) are in Table 1.

Table 1. Webb's path analysis results compared to two separate RC models.

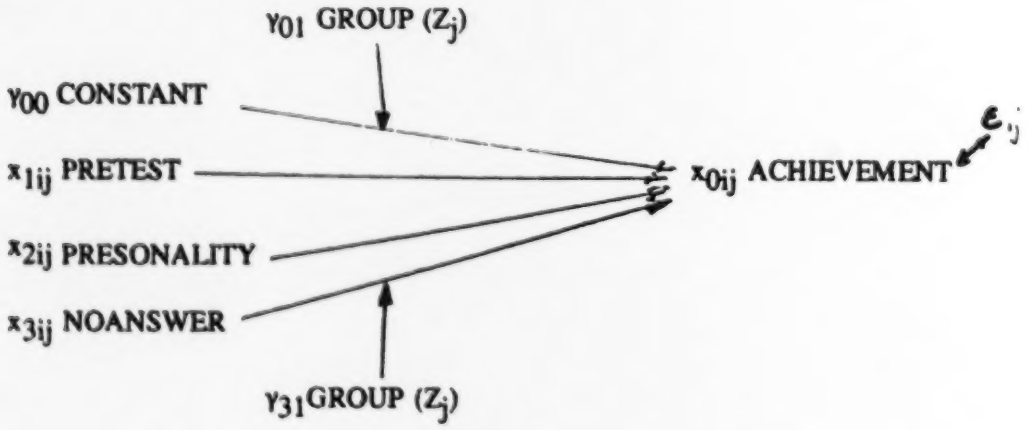
Path	Webb (1982, p.480) Coefficient	(S.E.)	Two RC models (1) and (2) Coefficient	(S.E.)
Path 1 Pretest	2.69	(0.46)	2.79	(0.46)
Path 2 Personality	-0.10	(0.03)	-0.06	(0.04)
Path 3 NoAnswer	-3.84	(0.62)	-3.68	(0.61)
Path 4 Group	-0.62	(0.21)	-0.52	(0.21)

Table 1 shows that the main difference between the two models is only in the magnitude of the coefficient for Personality. In Webb's results it is significant, while it is not in the RC model. As Webb reports (1982, p. 480) the significance test of the regression coefficients would be liberal if residuals within groups are positively correlated. "Such a result could lead to overinterpretation of marginal significant effects in the model". For the remainder the coefficients and the respective standard errors are very similar. Both analyses support Webb's hypotheses that "Interaction in the group may not influence achievement directly,..., but may instead influence another variable that affects achievement directly" (l.c. p. 479)

4.2. A single RC model

The previous analysis stays close to Webb's path analysis. The next model allows 'Group' to have a direct influence on achievement, instead of an indirect path via Noanswer. This goes against Webb's conclusion (l.c.p.483) that if small groups are included as a source of variance no significant differences are observed between uniform-ability and mixed-ability groups in the present study. By using a RC model, we introduce small groups as a sources of variation. In the RC model in Figure 3 the only dependent variable is achievement, predicted by personality, pre test and NoAnswer. At the macro level the variable for group homogeneity is introduced (Z_j). For this model we assume that the random intercept and the random slope for NoAnswer are both functions of Z_j (see Figure 3). In Webb's pathmodel one of these effects, the direct effect of Z_j on achievement, (which is the interaction of Z with the intercept or constant in Figure 3) is set to zero.

Figure 3.



See equations (3a) through (3d) for above model.

$$x_{0ij} = \alpha_j + \beta_1 x_{1ij} + \beta_2 x_{2ij} + \beta_{3j} x_{3ij} + \varepsilon_{ij} \qquad (3a)$$

where

$$\alpha_j = \gamma_{00} + \gamma_{01} Z_j + \delta_{0j}, \qquad (3b)$$

and

$$\beta_{3j} = \gamma_{30} + \gamma_{31} Z_j + \delta_{3j}, \qquad (3c)$$

In (3b) we assume an interaction of the group characteristic Z_j (mixed versus uniform grouping) with the intercept, while in (3c) we assume a cross level interaction of Z_j and the student variable NoAnswer(x_{3ij}). Adding and substituting (3b and 3c) into (3a) gives (3d),

$$x_{ij0} = \gamma_{00} + \gamma_{01} Z_j + \beta_1 x_{1ij} + \beta_2 x_{2ij\,j} + \gamma_{30} x_{3ij} + \gamma_{31} Z_j x_{3ij} + (\varepsilon_{ij} + \delta_{3j} x_{3ij} + \delta_{0j}) \qquad (3d)$$

Result are in first column of Table 2.

Table 2. The random and fixed model compared

Variables		Random Model Coefficients	(S.E.)		Fixed Model Coefficient	(S.E.)
	Intercept	14.51			15.77	(0.44)
Z_j	Group	-6.04	(2.70)	x_{4i}	-5.64	(2.47)
x_{1ij}	Pretest	2.63	(0.43)	x_{1i}	2.61	(0.44)
x_{2ij}	Personality	0.27	(0.21)	x_{2i}	0.20	(0.22)
x_{3ij}	NoAnswer	-5.15	(0.90)	x_{3i}	-5.18	(0.95)
Z_j/x_{3ij}	Interaction	2.83	(1.16)	x_{4i}/x_{3i}	2.64	(1.23)
Variance components		Sigma	(SE)			
Gr Mean		2.91	(0.77)		na	
NoAnswer		0.86	(0.74)*		na	
Deviance		611.11			615.41	
Residual variance		27.35			35.61	

* Note that the slope is no longer significantly random. The variability between groups of the relation NoAnswer (X) , Achievement (Y) is explained at group level by group-heterogeneity. The variability across groups was significant in a model without second level variable and without interaction. We found a sigma for NoAnswer of 2.19 with a SE of (0.62). The variation of the intercept (Gr. Mean in above table) remained the same across models with and without a random slope for NoAnswer.

Personality again is not significant, while the group effect and the interaction of group and NoAnswer are both significant, although of opposite sign. The negative group effect of -6.04 means that the lowest group (uniform ability coded as 0) has the highest achievement, while the interaction with Noanswer has a positive effect, meaning that in the higher group (mixed ability ability coded as 1) Noanswer has a positive effect on achievement. In fact this positive effect counteracts the overall negative effect of NoAnswer, which is -5.15, but only for mixed ability groups. The significant and positive cross level interaction of 2.83 supports Webb's findings, that the negative effect of NoAnswer on achievement is more detrimental in uniform ability groups (coded as 0) than in mixed ability groups (coded 1). Webb (1982) reports in the discussion part of her paper that her model failed to show a significant difference in achievement between uniform-ability and mixed-ability groups when the small groups were taken into account. Webb's assumption is (l.c. p.483) that "If the analysis in the present study had not included small groups as a factor, the difference between uniform-ability and mixed-ability groups would have been statistically significant". Her observation that different outcomes are the result of different procedures is clearly demonstrated in above analysis where we find a significant different effect on achievement across groups. The two effects of 'Group' in Figure 4 have clearly opposite effects for uniform ability groups. The direct and negative effect of 'Group' shows that uniform ability groups (coded 0) have higher overall predicted achievement than mixed ability groups (coded 1). But the interaction shows that the detrimental effect of NoAnswer is also higher in uniform groups. The RC model detects this direct effects of grouping on achievement, where Webb's traditional model could not. For the same reason an overall conclusion based on Webb's fixed model is quite different from one based on the random model. A possible conclusion based on Webb's (1982) analysis is, that heterogeneous groups produce higher achievements than homogeneous groups, because the variable 'NoAnswer' has a lower negative impact in the first. A possible conclusion based on our analysis is, that homogeneous groups produce overall higher achievement, which is lowered by a stronger detrimental effect of NoAnswer (see interaction = 2.83) in the same type of group.

For the purpose of comparing estimates obtained by the RC model with estimates produced by a fixed linear model, we added the results of the latter to Table 2. The first column shows results obtained with a RC model with random intercept, and random slope for NoAnswer. The second column show the results of a fixed intercept and fixed slope model, which is a traditional fixed effect multiple regression model. The subscript j disappears for the student level variables in the fixed model, since group membership is not taken into account. The group variable x_{4i} in the second model (a new notation for Z_j) is treated in the fixed effect model as a student level variable, hence the subscript i instead of j in x_{4i}.

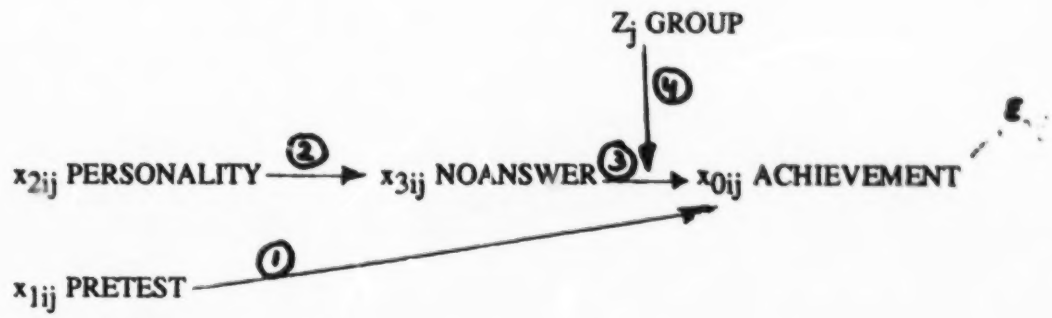
The differences between the fixed and random effect model in Table 2 are again minor. The standard error of the group variable is larger in the random model, since the last model uses the appropriate number of observations (which is equal to 35 the number of groups instead of 96 the number of students). All other estimates show that the RC model is slightly more powerful, since standard errors are smaller and values for the regression estimates are larger. The random model explains more variance, which is shown in a residual variance of 27.35 in the RC model, while the fixed model's residual variance is 35.61.

In above model we reported the contradictory effects of Group, either negative or positive. Before we make any interpretations based on these results we fit a RC path model, in which no constant(s) is (are) present and no direct effect of 'Group' on achievement. Only student level variables are part of the model. The group variable enters the model only as an interaction variable, interacting with the variable NoAnswer. The next analysis shows the result of such a model (based on De Leeuw and Kreft, 1992).

5. A multilevel path model.

Our research hypothesis is translated in a multilevel path model in Figure 4. This model does not replicate exactly the analysis as reported in Webb (1982), since the group variable is introduced solely as an interacting variable, with no direct path to the dependent variable or any other variable in the model. Group effects are assumed to work through relations between student variables by facilitating or suppressing these effects.

Figure 4.



The micro level equations of the model in Figure 4 are in (4a) and (5) and the macro equation in (4b). The first equation is in (4a) the second equation is in (5).

$$\underline{x}_{0ij} = \beta_{01}x_{1ij} + \underline{\beta}_{03j}\underline{x}_{3ij} + \varepsilon_{0ij} \qquad (4a)$$

where

$$\beta_{03j} = \gamma_{30} + \gamma_{31} Z_j + \delta_{3j} \qquad (4b)$$

Substitution (4b) in (4a) results in (4c)

$$\underline{x}_{0ij} = \beta_{01} x_{1ij} + (\gamma_{30} + \gamma_{31} Z_j + \delta_{3j}) \underline{x}_{3ij} + \varepsilon_{0ij} \qquad (4c)$$

Multiplication and rearranging gives (4d)

$$\underline{x}_{0ij} = \beta_{01} x_{1ij} + \gamma_{30} \underline{x}_{3ij} + \gamma_{31} Z_j \underline{x}_{3ij} + (\delta_{3j} \underline{x}_{3ij} + \varepsilon_{0ij}) \qquad (4d)$$

The second path is

$$\underline{x}_{3ij} = \beta_{32} x_{2ij} + \varepsilon_{3ij} \qquad (5)$$

The results are in Table 3

Table 3. Path analysis results

Path	Coefficient	(S.E.)	
Path 1 Pretest	2.80	(0.46)	
Path 2 Personality	-0.06	(0.04)	n.s.
Path 3 NoAnswer	-4.13	(0.82)	
Path 4 Interaction Group/NoAnswer	0.63	(0.73)	n.s.

Table 3 shows no effect of Group. Since Group is only represented in the model as an interaction with the path from NoAnswer to Achievement, the non significant coefficient for Intercation shows the absence of a group effect. Combining this result with the results of the RC model in Table 2, we may conclude that Group has two effects which counteract each other. Hence no significant effects in our ML pathmodel.

6. Discussion.

Our reanalysis of the Webb data shows that the estimates based on the RC models give very similar answers to fixed effects models when the same variables are used. The 'borrowing of strengths' in the Empirical Bayesian estimation procedure shrinks unreliable estimates

toward the overall estimate (the 'mean intercept or the 'mean' slope estimates), which are very close to the Least Squares estimates of the fixed effect linear model. Although Webb's study ignored group membership of students as well as the intra group correlation, while the RC models did not, the results over comparable models lead to the same conclusions. The difference between Webb's analysis and ours is not in the value of the regression coefficients, but is of another nature. Our conclusions differ from Webb's in relation to the intra class correlation and in relation with an overall 'Group' effect. Webb tested for intra class correlation by using a hierarchical analysis of variance on residuals of groups and showed that the groups were not significantly different. Based on this result Webb concludes that no intra class correlation exists. Our analysis shows a significant intra-group correlation of r= 0.23 and a significant group level error variance, hence we conclude that the intra class correlation is significant. Webb concludes that the effect of 'Group' is not direct, but mediated through NoAnswer. Our findings are that 'Group' has a direct effect on achievement as well. Conclusions based on results obtained by the model in Figure 3 are different from Webb's conclusions. We find that homogeneous groups have higher predicted achievement, while Webb concludes that homogeneous groups are suffering from the detrimental effect of the variable 'NoAnswer", and thus produce lower achievement. The outcome of the ML pathmodel shows no significant effects of the variable 'Group'.

Another favorable consequence of the choice of a RC model is that it offers many ways to analyze the data. The RC model invites the researcher to think in patterns of interaction, especially of cross-level interaction. Established variation at group level can be easily modeled in RC models as functions of group characteristics. As in our study, the variable 'Group' (homogeneity versus mixed ability grouping) is introduced twice, as a cross level interaction, explaining some of the group level variation in intercept and slope of NoAnswer. Given the goal that "The study of interaction in cooperative small groups are designed to explain Aptitude X Treatment interaction (ATI, see Webb, 1982, p.475), it follows that choosing a RC model is a logical choice in small group research.

The results over fixed and random models in the Tables 1 and 2 show that RC models produce comparable estimates, even when using small group data (see also Braun, 1989 for the stability of empirical Bayesian estimations). Since the values for the parameter estimates are close, we need other arguments to prefer RC models over the well known and established fixed effect linear models. In general we can argue that the assumptions of the random model are more realistic. In RC models we assume intra class correlation and random instead of fixed number of treatments. Ignoring these assumptions by choosing a fixed effect model may lead to "overinterpretation of marginal effects in the model" (Webb, 1982, p.480).

Taken all arguments together we conclude that the choice of a RC model in small group analysis is warranted for theoretical and for practical reasons. Since the sample design is nested (small groups within larger groups) the analysis model should reflect this (Cochran, 1977). Also when research is aimed at testing variable relations across groups (as in ATI research) RC models are a good choice, and in case of small group data, the only option left. RC models are also preferred for practical reasons, because they provide incentives for researchers to go beyond the traditional way of data analysis, to check for interactions between group level and individual level characteristics, to explain why different effects for different groups exist.

References.

Aitkin and Longford (1986). Statistical modelling issues in school effectiveness studies. *Journal of the Royal Statistical Society,* 149, 1-43.

Bassiri, Dina (1990). *Large and small sample properties of maximum likelihood estimates for the hierarchical linear model.* Paper presented at the Annual meeting of the AERA, Boston.

Braun, Henry, I. (1989). *Empirical Bayes Methods: a tool for exploratory analysis.* In: Multilevel Analysis of Educational Data. R.Darrell Bock (ed.). San Diego, Academic Press.

Burstein, L., Linn, R.L., and Capell, F.J. (1978). Analyzing multilevel data in the presence of heterogeneous within-class regressions. *Journal of Educational Statistics*, 3, 347-383.

Cochran, W. G. (1977). *Sampling Techniques (third edition).* New York, Wiley and Sons.

Cronbach, L.J. and Webb, N. (1975). Between class and within class effects in a reported aptitude X treatment interaction: a reanalysis of a study by G.L. Anderson. *Journal of Educational Psychology*, 67, p. 717-724.

De Leeuw Jan, and Kreft Ita G.G. (1986). Random coefficient models for multilevel analysis. *Journal of Educational Statistics*, 11, 1, 57-86.

DeLeeuw, Jan and Ita G.G. Kreft (forth coming). Multilevel Pathmodels

GENMOD (1988). *Manual for GENMOD.* by W.M. Mason, A.F. Anderson, and N. Hayat. Ann Arbor: Population Studies Center, University of Michican

HLM. (1988) *An introduction to HLM: Computer program and user' guide.* Developed by A.S. Bryk, S.W. Raudenbush, M. Seltzer, and R.T. Congdon. (1988)

Kim, Kyung-Sung (1990). *Multilevel Data Analysis: A comparative examination of analytical alternatives.* Dissertation, University of California, Los Angeles.

Kreft,Ita G.G., De Leeuw Jan, and Kim Kyung-Sung (1990). *Comparing four different statistical packages for hierarchical linear regression.* GENMOD, HLM, ML2 AND

VARCL.

Mason, Wong and Entwisle (1984). Contextual analysis through the multilevel linear model. *Sociological Methodology*, 72-103.

ML3 (1989). *ML3 Software for two-level analysis user'guide.* Developed by Rabash, J., Prosser, R. and Goldstein, H. at the Institute of Education, the University of London.

Raudenbush, Stephen, H.and Bryk Anton, S.,(1986). *A hierarchical model for studying school effects. Sociology of Education, 59, 1-17.*

Raudenbush, Stephen, H. and Willms, Douglas, J. (eds.) (1991). The Organisation of schooling and its methodological implications, p. 1-13. in: *Schools, Classrooms, and pupils. International Studies of Schooling from a multilevel perspective.* San Diego, Academic Press.

Snijders Tom, A.B. and Bosker Roel, J. (1992) Standard errors and sample sizes for two-level research. Journal of Educational Statistics (in press).

VARCL (1986). Program developed at the center for applied Statistics, Lancaster University by Longford, N.T.

Webb, Noreen M. (1982). Group composition, group interaction and achievement in cooperative small groups. Journal of Educational Psychology, 74, 475-484.

Webb, Noreen M. (1984). Stability of Small group Interaction and Achievement Over Time. *Journal of Educational Psychology*, 76, 2, 211-224.

Webb, Noreen M. and Cullian Linda K. (1983). Group Interaction and Achievement in Small Groups: Stability Over Time. *American Educational Research Journal*, 20, 3, p.411-423.